P P SAVANI UNIVERSITY

Third Semester of B. Tech. Examination
Nov-Dec 2021
SESH2040 Discrete Mathematics

03.12.2021, Friday Time: 09:00 a.m. To 11:30 a.m. Maximum Marks: 60

Instructions:
1. The question paper comprises of two sections.
2. Section I and II must be attempted in same answer sheets.
3. Make suitable assumptions and draw neat figures wherever required.
4. Use of scientific calculator is allowed.

SECTION – I

Answer the following: (Attempt any Six)

Q - 1 List the ordered pairs in the relation R from $A = \{0,1,2,3,4\}$ to $B = \{0,1,2,3\}$, where $(a, b) \in R$ if and only if [05]

    (a) $a = b$ (b) $a + b = 4$ (c) $a > b$

    (d) $a|b$ (e) $\gcd(a, b) = 1$

Q - 2 Determine whether the relation R on the set of all real numbers is reflexive, symmetric, antisymmetric, and/or transitive, where $(x, y) \in R$ if and only if [05]

    (a) $x \neq y$.   (b) $xy \geq 1$ (c) $x = y + 1 \ or \ x = y - 1$

    (d) x is a multiple of y.   (e) x and y are both negative or both nonnegative.

Q - 3 Draw the Hasse diagram for the "less than or equal to" relation on $\{0, 2, 5, 10, 11, 15\}$. [05]

Q - 4 Consider the lattice $D_{60} = \{1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60\}$, the divisors of 60 ordered by divisibility. Draw the diagram of D_{60}

**Q - 5** Consider the bounded lattice L. [05]
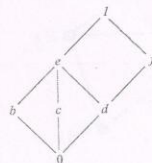
    (a) Find complements of e and f, if they exist.

    (b) Is L distributive? Complemented?

Q - 6 Let $G = \{\pm 1, \pm i, \pm j, \pm k\}$. Then show that G is a non-abelian group under the usual multiplication '$*$'. [05]

    Multiplication Rule:

    $i.i = j.j = k.k = -1; i.j = -j.i = k; j.k = -k.j = i; k.i = -i.k = j$

Q - 7 If G is the set of all ordered pairs (a, b) where $a(\neq 0)$ and b are real the binary operation $*$ on G is defined by $(a, b) * (c, d) = (ac, bc + d)$, show that $(G,*)$ is non-abelian group. Show also that the subset H of all those elements of G which are of the form $(1, b)$ is a subgroup of G. [05]

Q - 8 Prove that all positive integers are not a group under addition. [05]

SECTION – II

Answer the following: (Attempt any Six)

Q - 1 Which of these sentences are propositions? What are the truth values of those that are propositions? [05]

    (a) Boston is the capital of Massachusetts.   (c) $2 + 3 = 5$

    (b) Miami is the capital of Florida.   (d) $5 + 7 = 10$ (e) $x + 2 = 11$

Q - 2 Prove that $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$. [05]

Q - 3 Find a shortest route (in distance) between computer centers in each of these pairs of cities [05] in the communications network shown in Figure.
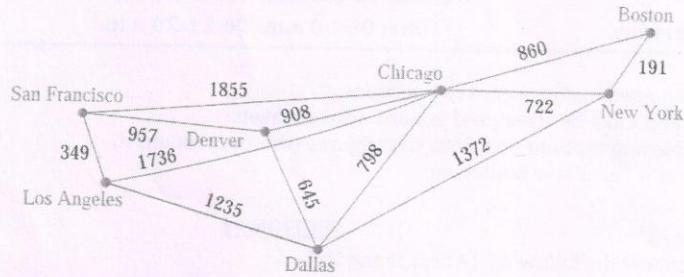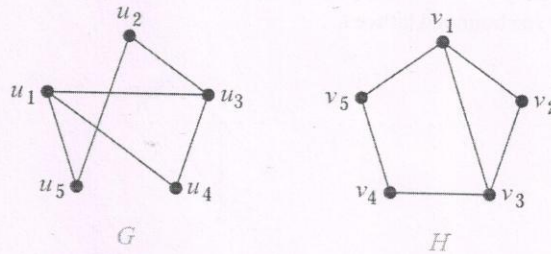

(a) Boston and Los Angeles
(b) New York and San Francisco

Q - 4 Determine whether the given graph is connected. [05]

(a) (b)

Q - 5 Determine whether the graphs G and H shown in Figure are isomorphic. [05]

G H

Q - 6 Draw five rooted tree and write the height and level for all. [05]

Q - 7 In the rooted tree T (with root a) shown in Figure, find the parent of c, the children of g, the siblings of h, all ancestors of e, all descendants of b, all internal vertices, and all leaves. What is the subtree rooted at g? [05]
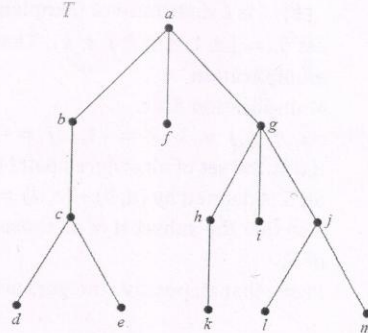
Q - 8 Draw the five examples of the $m - ary$ tree and Full (Strict) $m - ary$ tree. [05]

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